nag_mv_fac_score (g03ccc)

1. Purpose

nag_mv_fac_score (g03ccc) computes factor score coefficients from the result of fitting a factor analysis model by maximum likelihood as performed by nag_mv_factor (g03cac).

2. Specification

```
#include <nag.h>
#include <nagg03.h>

void nag_mv_fac_score(Nag_FacScoreMethod method, Nag_FacRotation rotate,
           Integer nvar, Integer nfac, double fl[], Integer tdfl, double psi[],
           double e[], double r[], Integer tdr, double fs[], Integer tdfs,
           NagError *fail)
```

3. Description

A factor analysis model aims to account for the covariances among p variables, observed on n individuals, in terms of a smaller number, k, of unobserved variables or factors. The values of the factors for an individual are known as factor scores. nag_mv_factor (g03cac) fits the factor analysis model by maximum likelihood and returns the estimated factor loading matrix, Λ, and the diagonal matrix of variances of the unique components, Ψ. To obtain estimates of the factors, a p by k matrix of factor score coefficients, Φ, is formed. The estimated vector of factor scores, $\hat{f}$, is then given by:

$$\hat{f} = x^T \Phi,$$

where x is the vector of observed variables for an individual.

There are two commonly used methods of obtaining factor score coefficients.

The regression method:

$$\Phi = \Psi^{-1}\Lambda(I + \Lambda^T\Psi^{-1}\Lambda)^{-1},$$

and Bartlett's method:

$$\Phi = \Psi^{-1}\Lambda(\Lambda^T\Psi^{-1}\Lambda)^{-1}.$$

See Lawley and Maxwell (1971) for details of both methods. In the regression method as given above, it is assumed that the factors are not correlated and have unit variance; this is true for models fitted by nag_mv_factor (g03cac). Further, for models fitted by nag_mv_factor (g03cac),

$$\Lambda^T\Psi^{-1}\Lambda = \Theta - I,$$

where Θ is the diagonal matrix of eigenvalues of the matrix S^*, as described in nag_mv_factor (g03cac).

The factors may be orthogonally rotated using an orthogonal rotation matrix, R, as computed by nag_mv_orthomax (g03bac). The factor scores for the rotated matrix are then given by ΛR.

4. Parameters

method

Input: indicates which method is to be used to compute the factor score coefficients.

If **method** = **Nag_FacScoreRegsn**, then the regression method is used.

If **method** = **Nag_FacScoreBart**, then Bartlett's method is used.

Constraint: **method** = **Nag_FacScoreRegsn** or **Nag_FacScoreBart**.

rotate

Input: indicates whether a rotation is to be applied.

If **rotate** = **Nag_FacRotate**, then a rotation will be applied to the coefficients and the rotation matrix, R, must be given in **r**.
If **rotate** = **Nag_FacNoRotate**, then no rotation is applied.

Constraint: **rotate** = **Nag_FacRotate** or **Nag_FacNoRotate**.

nvar

Input: the number of observed variables in the factor analysis, p.

Constraint: **nvar** $\geq$ **nfac**.

nfac

Input: the number of factors in the factor analysis, k.

Constraint: **nfac** ≥ 1.

fl[nvar][tdfl]

Input: the matrix of unrotated factor loadings, Λ, as returned by nag_mv_factor (g03cac).

tdfl

Input: the last dimension of the array **fl** as declared in the calling program.
Constraint: **tdfl** $\geq$ **nfac**.

psi[nvar]

Input: the diagonal elements of Ψ, as returned by nag_mv_factor (g03cac).

Constraint: **psi**$[i-1] > 0.0$, for $i = 1, 2, \ldots, p$.

e[nvar]

Input: the eigenvalues of the matrix S^*, as returned by nag_mv_factor (g03cac).

Constraint: **e**$[i-1] > 1.0$, for $i = 1, 2, \ldots, p$.

r[nfac][tdr]

Input: if **rotate** = **Nag_FacRotate**, then **r** must contain the orthogonal rotation matrix, R, as returned by nag_mv_orthomax (g03bac). If **rotate** = **Nag_FacNoRotate** then **r** need not be set.

tdr

Input: the last dimension of the array **r** as declared in the calling program.
Constraint: if **rotate** = **Nag_FacRotate** then **tdr** $\geq$ **nfac**.

fs[nvar][tdfs]

Output: the matrix of factor score coefficients, Φ. **fs**$[i-1][j-1]$ contains the factor score coefficient for the jth factor and the ith observed variable, for $i = 1, 2, \ldots, p$; $j = 1, 2, \ldots, k$.

tdfs

Input: the last dimension of the array **fs** as declared in the calling program.
Constraint: **tdfs** $\geq$ **nfac**.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_BAD_PARAM

On entry, parameter **method** had an illegal value.
On entry, parameter **rotate** had an illegal value.

NE_INT_ARG_LT

On entry, **nfac** must not be less than 1: **nfac** = $\langle$*value*$\rangle$.

NE_2_INT_ARG_LT

On entry, **nvar** = ⟨*value*⟩ while **nfac** = ⟨*value*⟩.
These parameters must satisfy **nvar** ≥ **nfac**.
On entry, **tdfl** = ⟨*value*⟩ while **nfac** = ⟨*value*⟩.
These parameters must satisfy **tdfl** ≥ **nfac**.
On entry, **tdfs** = ⟨*value*⟩ while **nfac** = ⟨*value*⟩.
These parameters must satisfy **tdfs** ≥ **nfac**.

NE_2_INT_ARG_ENUM_CONS

On entry, **tdr** = ⟨*value*⟩ while **nfac** = ⟨*value*⟩ and **rotate** = **Nag_FacRotate**.
These parameters must satisfy **tdr** ≥ **nfac** when **rotate** = **Nag_FacRotate**.

NE_REAL_ARRAY_INPUT

On entry, **e**[⟨*value*⟩] = ⟨*value*⟩.
Constraint: **e**[⟨*value*⟩] > 1.0.
On entry, **psi**[⟨*value*⟩] = ⟨*value*⟩.
Constraint: **psi**[⟨*value*⟩] > 0.0.

NE_ALLOC_FAIL

Memory allocation failed.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6. Further Comments

To compute the factor scores using the factor score coefficients, the values for the observed variables first need to be standardized by subtracting the sample means and, if the factor analysis is based upon a correlation matrix, dividing by the sample standard deviations. This may be performed using nag_mv_z_scores (g03zac). The standardized variables are then post-multiplied by the factor score coefficients.

If principal component analysis is required, the routine nag_mv_prin_comp (g03aac) computes the principal component scores directly. Hence, the factor score coefficients are not needed.

6.1. Accuracy

Accuracy will depend on the accuracy requested when computing the estimated factor loadings using nag_mv_factor (g03cac).

6.2. References

Lawley D N and Maxwell A E (1971) *Factor Analysis as a Statistical Method* Butterworths (2nd Edition).

7. See Also

nag_mv_canon_corr (g03adc)
nag_mv_factor (g03cac)
nag_mv_z_scores (g03zac)
nag_mv_orthomax (g03bac)