

## nag\_regsn\_mult\_linear\_newyvar (g02dgc)

### 1. Purpose

**nag\_regsn\_mult\_linear\_newyvar (g02dgc)** calculates the estimates of the parameters of a general linear regression model for a new dependent variable after a call to **nag\_regsn\_mult\_linear (g02dac)**.

### 2. Specification

```
#include <nag.h>
#include <nagg02.h>

void nag_regsn_mult_linear_newyvar(Integer n, double wt[], double *rss,
    Integer ip, Integer rank, double cov[], double q[], Integer tdq,
    Boolean svd, double p[], double y[], double b[], double se[],
    double res[], double com_ar[], NagError *fail)
```

### 3. Description

**nag\_regsn\_mult\_linear\_newyvar** uses the results given by **nag\_regsn\_mult\_linear (g02dac)** to fit the same set of independent variables to a new dependent variable.

**nag\_regsn\_mult\_linear (g02dac)** computes a  $QR$  decomposition of the matrix of  $p$  independent variables and also, if the model is not of full rank, a singular value decomposition (SVD). These results can be used to compute estimates of the parameters for a general linear model with a new dependent variable. The  $QR$  decomposition leads to the formation of an upper triangular  $p$  by  $p$  matrix  $R$  and an  $n$  by  $n$  orthogonal matrix  $Q$ . In addition the vector  $c = Q^T y$  (or  $Q^T W^{1/2} y$ ) is computed. For a new dependent variable,  $y_{\text{new}}$ , **nag\_regsn\_mult\_linear\_newyvar** computes a new value of  $c = Q^T y_{\text{new}}$  or  $Q^T W^{1/2} y_{\text{new}}$ .

If  $R$  is of full rank, then the least-squares parameter estimates,  $\hat{\beta}$ , are the solution to:  $R\hat{\beta} = c_1$ , where  $c_1$  is the first  $p$  elements of  $c$ .

If  $R$  is not of full rank, then **nag\_regsn\_mult\_linear (g02dac)** will have computed the SVD of  $R$ ,

$$R = Q_* \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix} P^T$$

where  $D$  is a  $k$  by  $k$  diagonal matrix with non-zero diagonal elements,  $k$  being the rank of  $R$ , and  $Q_*$  and  $P$  are  $p$  by  $p$  orthogonal matrices. This gives the solution

$$\hat{\beta} = P_1 D^{-1} Q_{*1}^T c_1$$

$P_1$  being the first  $k$  columns of  $P$ , i.e.,  $P = (P_1 P_0)$  and  $Q_{*1}$  being the first  $k$  columns of  $Q_*$ . Details of the SVD are made available by **nag\_regsn\_mult\_linear (g02dac)** in the form of the matrix  $P^*$ :

$$P^* = \begin{pmatrix} D^{-1} P_1^T \\ P_0^T \end{pmatrix}.$$

The matrix  $Q_*$  is made available through the **com\_ar** parameter of **nag\_regsn\_mult\_linear (g02dac)**.

In addition to parameter estimates, the new residuals are computed and the variance-covariance matrix of the parameter estimates are found by scaling the variance-covariance matrix for the original regression.

### 4. Parameters

**n**

Input: the number of observations,  $n$ .

Constraint:  $n \geq 2$ .

**wt[n]**

Input: if weighted estimates are required then **wt** must contain the weights to be used in the weighted regression. Otherwise **wt** need not be defined and may be set to the null pointer **NULL**, i.e., (double \*) 0.

If  $\text{wt}[i] = 0.0$ , then the  $i$ th observation is not included in the model, in which case the effective number of observations is the number of observations with non-zero weights. The values of **res** and **h** will be set to zero for observations with zero weights.

If **wt** = **NULL**, then the effective number of observations is  $n$ .

Constraint: **wt** = **NULL** or  $\text{wt}[i] \geq 0.0$ , for  $i = 0, 1, \dots, n - 1$ .

**rss**

Input: the residual sum of squares for the original dependent variable.

Output: the residual sum of squares for the new dependent variable.

**ip**

Input: the number  $p$  of independent variables in the model (including the mean if fitted).

Constraint:  $1 \leq \text{ip} \leq n$ .

**rank**

Input: the rank of the independent variables, as given by nag\_regsn\_mult\_linear (g02dac).

Constraint: **rank** > 0 and if **svd** = **FALSE**, **rank** = **ip** otherwise **rank** ≤ **ip**.

**cov[ip\*(ip+1)/2]**

Input: the covariance matrix of the parameter estimates as given by nag\_regsn\_mult\_linear (g02dac).

Output: the upper triangular part of the variance-covariance matrix of the **ip** parameter estimates given in **b**. They are stored packed by column, i.e., the covariance between the parameter estimate given in  $\mathbf{b}[i]$  and the parameter estimate given in  $\mathbf{b}[j]$ ,  $j \geq i$ , is stored in  $\text{cov}[j(j+1)/2 + i]$  for  $i = 0, 1, \dots, \text{ip} - 1$  and  $j = i, i + 1, \dots, \text{ip} - 1$ .

**q[n][tdq]**

Input: the results of the  $QR$  decomposition as returned by nag\_regsn\_mult\_linear (g02dac).

Output: the first column of **q** contains the new values of  $c$ , the remainder of **q** will be unchanged.

**tdq**

Input: the second dimension of the array **q** as declared in the function from which nag\_regsn\_mult\_linear\_newyvar is called.

Constraint: **tdq** ≥ **ip** + 1.

**svd**

Input: indicates if a singular value decomposition was used by nag\_regsn\_mult\_linear (g02dac).

If **svd** = **TRUE**, a singular value decomposition was used by nag\_regsn\_mult\_linear (g02dac).

If **svd** = **FALSE**, a singular value decomposition was not used by nag\_regsn\_mult\_linear (g02dac).

**p[2\*ip+ip\*ip]**

Input: details of the  $QR$  decomposition and SVD, if used, as returned in array **p** by nag\_regsn\_mult\_linear (g02dac).

If **svd** = **FALSE**, only the first **ip** elements of **p** are used, these will contain the zeta values for the  $QR$  decomposition.

If **svd** = **TRUE**, the first **ip** elements of **p** will contain the zeta values for the  $QR$  decomposition and the next **ip** elements of **p** contain singular values. The following **ip** by **ip** elements contain the matrix  $P^*$  stored by rows.

**y[n]**

Input: the new dependent variable  $y_{\text{new}}$ .

**b[ip]**

Output:  $\mathbf{b}[i]$ ,  $i = 0, 1, \dots, \text{ip} - 1$  contain the least-squares estimates of the parameters of the regression model,  $\hat{\beta}$ .

- se[ip]**  
Output: **se**[ $i$ ],  $i = 0, 1, \dots, \mathbf{ip} - 1$  contain the standard errors of the **ip** parameter estimates given in **b**.
- res[n]**  
Output: the residuals for the new regression model.
- com\_ar[5\*(ip-1)+ip\*ip]**  
Input: if **svd** = **TRUE**, **com\_ar** must be unaltered from the previous call to `nag_regsn_mult_linear` (g02dac).
- fail**  
The NAG error parameter, see the Essential Introduction to the NAG C Library.

## 5. Error Indications and Warnings

### NE\_INT\_ARG\_LT

On entry, **ip** must not be less than 1: **ip** =  $\langle value \rangle$ .

### NE\_INT\_ARG\_LE

On entry, **rank** must not be less than or equal to 0: **rank** =  $\langle value \rangle$ .

### NE\_2\_INT\_ARG\_LT

On entry, **tdq** =  $\langle value \rangle$  while **ip** + 1 =  $\langle value \rangle$ . These parameters must satisfy **tdq**  $\geq$  **ip** + 1.  
On entry, **n** =  $\langle value \rangle$  while **ip** =  $\langle value \rangle$ . These parameters must satisfy **n**  $\geq$  **ip**.

### NE\_REAL\_ARG\_LE

On entry, **rss** must not be less than or equal to 0.0: **rss** =  $\langle value \rangle$ .

### NE\_REAL\_ARG\_LT

On entry, **wt**[ $\langle value \rangle$ ] must not be less than 0.0: **wt**[ $\langle value \rangle$ ] =  $\langle value \rangle$ .

### NE\_SVD\_RANK\_NE\_IP

On entry, the Boolean variable, **svd**, is **FALSE** and **rank** must be equal to **ip**: **rank** =  $\langle value \rangle$ , **ip** =  $\langle value \rangle$ .

### NE\_SVD\_RANK\_GT\_IP

On entry, the Boolean variable, **svd**, is **TRUE** and **rank** must not be greater than **ip**: **rank** =  $\langle value \rangle$ , **ip** =  $\langle value \rangle$ .

## 6. Further Comments

The values of the leverages,  $h_i$ , are unaltered by a change in the dependent variable so a call to `nag_regsn_std_resid_influence` (g02fac) can be made using the value of **h** from `nag_regsn_mult_linear` (g02dac).

### 6.1. Accuracy

The same accuracy as `nag_regsn_mult_linear` (g02dac) is obtained.

### 6.2. References

- Golub G H and Van Loan C F (1983) *Matrix Computations* Johns Hopkins University Press, Baltimore.
- Hammarling S (1985) The Singular Value Decomposition in Multivariate Statistics *ACM Signum Newsletter* **20** (3) 2–25.
- Searle S R (1971) *Linear Models* Wiley.

## 7. See Also

- `nag_regsn_mult_linear` (g02dac)  
`nag_regsn_std_resid_influence` (g02fac)

## 8. Example

A data set consisting of 12 observations with four independent variables and two dependent variables is read in. A model with all four independent variables is fitted to the first dependent variable by nag\_regsn\_mult\_linear (g02dac) and the results printed. The model is then fitted to the second dependent variable by nag\_regsn\_mult\_linear\_newyvar and those results printed.

### 8.1. Program Text

```

/* nag_regsn_mult_linear_newyvar(g02dgc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg02.h>

#define NMAX 12
#define MMAX 5
#define TDQ MMAX+1
#define TDXM MMAX

main()
{
    double  rss, tol;
    Integer i, ip, rank, j, m, n;
    double  df;
    Boolean svd;
    Nag_IncludeMean  mean;
    char  weight, meanc;
    double  b[MMAX], cov[MMAX*(MMAX+1)/2], h[NMAX], newy[NMAX],
    p[MMAX*(MMAX+2)], q[NMAX][MMAX+1], res[NMAX], se[MMAX],
    com_ar[5*(MMAX-1)+MMAX*MMAX], wt[NMAX], xm[NMAX][MMAX], y[NMAX];
    Integer  sx[MMAX];
    double  *wtptr;

    Vprintf("g02dgc Example Program Results\n");
    /* Skip heading in data file */
    Vscanf("%*[^\\n]");
    Vscanf("%ld %ld %c %c", &n, &m, &weight, &meanc);
    if (meanc=='m')
        mean = Nag_MeanInclude;
    else
        mean = Nag_MeanZero;
    if (n<=NMAX && m<MMAX)
    {
        if (weight=='w')
        {
            wtptr = wt;
            for (i=0; i<n; i++)
            {
                for (j=0; j<m; j++)
                    Vscanf("%lf", &xm[i][j]);
                Vscanf("%lf%lf%lf", &y[i], &wt[i], &newy[i]);
            }
        }
        else
        {
            wtptr = (double *)0;
            for (i=0; i<n; i++)
            {
                for (j=0; j<m; j++)
                    Vscanf("%lf", &xm[i][j]);
                Vscanf("%lf%lf", &y[i], &newy[i]);
            }
        }
        for (j=0; j<m; j++)

```

```

    Vscanf("%ld", &sx[j]);
    Vscanf("%ld", &ip);
    /* Set tolerance */
    tol = 0.00001e0;
    /* Fit initial model using g02dac */
    g02dac(mean, n, (double *)xm, (Integer)TDXM, m, sx, ip,
           y, wtptr, &rss, &df, b, se, cov, res, h, (double *)q,
           (Integer)(TDQ), &svd, &rank, p, tol, com_ar, NAGERR_DEFAULT);

    Vprintf("Results from g02dac\n\n");
    if (svd)
        Vprintf("Model not of full rank\n\n");
    Vprintf("Residual sum of squares = %12.4e\n", rss);
    Vprintf("Degrees of freedom = %3.1f\n\n", df);
    Vprintf("Variable   Parameter estimate   Standard error\n\n");
    for (j=0; j<ip; j++)
        Vprintf("%6ld%20.4e%20.4e\n", j+1, b[j], se[j]);
    Vprintf("\n");

    g02dgc(n, wtptr, &rss, ip, rank, cov, (double *)q, (Integer)(TDQ), svd, p,
           newy, b, se, res, com_ar, NAGERR_DEFAULT);

    Vprintf("\n");
    Vprintf("Results for second y-variable using g02dgc\n\n");
    Vprintf("Residual sum of squares = %12.4e\n", rss);
    Vprintf("Degrees of freedom = %3.1f\n\n", df);
    Vprintf("Variable   Parameter estimate   Standard error\n\n");
    for (j=0; j<ip; j++)
        Vprintf("%6ld%20.4e%20.4e\n", j+1, b[j], se[j]);
    Vprintf("\n");
}
else
{
    Vfprintf(stderr, "One or both of m and n are out of range:\n
m = %-3ld while n = %-3ld\n", m, n);
    exit(EXIT_FAILURE);
}
exit(EXIT_SUCCESS);
}
}

```

## 8.2. Program Data

```

g02dgc Example Program Data
 12 4   u m
1.0 0.0 0.0 0.0 33.63 63.0
0.0 0.0 0.0 1.0 39.62 69.0
0.0 1.0 0.0 0.0 38.18 68.0
0.0 0.0 1.0 0.0 41.46 71.0
0.0 0.0 0.0 1.0 38.02 68.0
0.0 1.0 0.0 0.0 35.83 65.0
0.0 0.0 0.0 1.0 35.99 65.0
1.0 0.0 0.0 0.0 36.58 66.0
0.0 0.0 1.0 0.0 42.92 72.0
1.0 0.0 0.0 0.0 37.80 67.0
0.0 0.0 1.0 0.0 40.43 70.0
0.0 1.0 0.0 0.0 37.89 67.0
 1 1 1 1 5

```

### 8.3. Program Results

g02dgc Example Program Results  
Results from g02dac

Model not of full rank

Residual sum of squares = 2.2227e+01  
Degrees of freedom = 8.0

Variable	Parameter estimate	Standard error
1	3.0557e+01	3.8494e-01
2	5.4467e+00	8.3896e-01
3	6.7433e+00	8.3896e-01
4	1.1047e+01	8.3896e-01
5	7.3200e+00	8.3896e-01

Results for second y-variable using g02dgc

Residual sum of squares = 2.4000e+01  
Degrees of freedom = 8.0

Variable	Parameter estimate	Standard error
1	5.4067e+01	4.0000e-01
2	1.1267e+01	8.7178e-01
3	1.2600e+01	8.7178e-01
4	1.6933e+01	8.7178e-01
5	1.3267e+01	8.7178e-01

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